ANNUAL

SHERWOOD THEORETICAL MEETING

APRIL 24-25, 1969

GATLINBURG, TENNESSEE

ABSTRACTS

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OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
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SESSION I - April 24, 9:00 a.m. - Chairman, J. M. Dawson

1. J. D. Callen and C. W. Horton, "Modified Negative Mass Instability".

2. H. L. Berk, L. D. Pearlstein, and G. M. Walters, "Interaction of Negative Mass and Loss Cone Modes".

3. L. D. Pearlstein and H. L. Berk, "Stabilization of Negative Energy Eigenmodes in Mirror Geometry".

4. Laurence S. Hall, "Pulse Propagation in Inhomogeneous Convectively Unstable Plasma".

5. R. E. Aamodt, "Particle Motion in the Presence of Multiple Finite Amplitude Harmonic Cyclotron Modes and the Non-Linear Dynamics of Resonant Loss-Cone Instabilities".

6. J. Fukai, S. Krishan, and E. G. Harris, "Wave-Wave Interactions, Wave Particle Scattering, and Explosive Instabilities".

7. C. K. Birdsall, J. A. Byers, D. Fuss, and M. Grewal, "Unstable Plasma Waves Propagating Perpendicular to a Magnetic Field: Small Amplitude Growth and Nonlinear Saturation from Computer Experiments".

8. E. Canobbio and G. Ichtchenko, "On the Reversibility of the Gyroresonance in Mirror Machines".

SESSION II - April 24, 2:00 p.m. - Chairman, R. L. Morse

1. S. Yoshikawa, "Toroidal Equilibria Including E x B and Parallel Plasma Flow".

2. T. E. Stringer, "Diffusion in Toroidal Plasmas with Radial Electric Field".

3. D. R. Dobrott, J. M. Greene, and J. L. Johnson, "Effect of Trapped Particles on Stellarator Equilibria".

4. N. K. Winsor, J. L. Johnson, and J. M. Dawson, "Numerical Simulation of Toroidal Low-β Confinement with a Fluid Model".

5. C. G. Smith, "Equilibrium Electric Fields in Axisymmetric Toroids".

6. Harold Grad and Harold Weitzner, "Critical β from Stellarator and Scyllac Expansions".

7. S. Fisher, B. Marder, and H. Weitzner, "Astron E. Layer Equilibria".

8. G. K. Morikawa and E. Rebhan, "Hydromagnetic Plasma Confinement in a Spherical Geometry".

SESSION III – April 25, 9:00 a.m. – Chairman, E. G. Harris

1. J. R. Thompson and W. E. Drummond, "Non-Linear Evolution of the Two Stream Instability".

2. C. W. Nielson and R. L. Morse, "Numerical Simulation of Two-Beam Plasmas".

3. G. Laval, R. Pellat, and A. Roux, "Non-Linear Damping of a Finite Amplitude Monochromatic Wave".

4. Burton D. Fried, Chuan Sheng Liu, and R. Z. Sagdeev, "On-Line Plasma Simulation".

5. D. O. Dickman, R. L. Morse, and C. W. Nielson, "Numerical Simulation of Firehose Instability".

6. D. C. Stevens, "Numerical Study of Finite β Drift Waves".

7. W. L. Kruer and J. M. Dawson, "A Trapped Particle Instability".

8. Barry Marder, "Compression of an Axially Symmetric Plasma with Non-Isotropic Pressure".

SESSION IV – April 25, 2:00 p.m. – Chairman, T. K. Fowler

1. H. Vernon Wong, "Non-Linear Evolution of a Single Electrostatic Electron-Ion Streaming Unstable Mode".

2. C. W. Horton and F. L. Hinton, "Amplitude Limitation and Transport for Collisional Drift Waves".

3. Thomas H. Dupree, "Time and Space Correlation Due to Wave-Particle Scattering".

4. F. L. Hinton and C. Oberman, "Electrical Conductivity of Plasma in a Spatially Inhomogeneous Magnetic Field".

5. J. P. Friedberg and J. A. Wesson, "Ion Drift Wave Instability".

6. Nicholas A. Krall and Ronald C. Davidson, "Vlasov Description of a Pure Electron Plasma in a Magnetic Field".

7. R. G. Bateman and M. D. Kruskal, "Dispersion Relation for Electron Oscillations".

8. P. Kaw and J. M. Dawson, "Laser Induced Anomalous Heating of a Plasma".

SESSION I

April 24, 9:00 a.m. - Chairman, J. M. Dawson

Modified Negative Mass Instability*

J. D. Callen and C. W. Horton
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The original studies of the modified negative mass instability[1, 2] (due to a weak axial variation in the magnetic field (B) with $T_\perp >> T_\parallel$ and characterized by a high density cut-off) have been extended to include the effects of thermal spreading in the ion velocity distribution, particle drifts, radial variations in B and particle mirroring (bounce effects). Thermal spreading of the pitch-angle ($\phi = v_\parallel / v_\perp$) distribution is found to introduce a low density instability threshold which exceeds the density for the onset of the absolute temperature-anisotropy instability[3] (another important instability in this regime) roughly when the mean pitch-angle becomes less than the thermal spread (FWHM) in the pitch-angle distribution. Consideration of the radial variation of B--including particle drifts and a new finite ion Larmor radius effect--shows that the modified negative mass instabilities having the lowest azimuthal mode numbers are stabilized in a sufficiently deep (radial) min-B well; for stability $\frac{3}{2} \frac{a_i^2}{R_p R_c} \gtrsim \frac{T_\parallel}{T_\perp}$ where a_i = ion Larmor radius, R_p = radius of plasma, and R_c = an average radius of curvature. Conversely, a radial "hill" in B enhances the instability. Bounce effects appear to stabilize the instability if $L/a_i \gtrsim 4T_\perp / T_\parallel$, where L is the scale-length of the axial variation of B.

1. J. F. Clarke and G. G. Kelley, Phys. Rev. Letters 21, 1041 (1968).
2. B. B. Kadomtsev and O. P. Pogutse, Paper CN-24/G-10, Novosibirsk Conf. (1968).
3. C. O. Beasley and J. G. Cordey, Plasma Physics 10, 411 (1968).

* Work performed under the auspices of the U. S. Atomic Energy Commission.

INTERACTION OF NEGATIVE MASS AND LOSS CONE MODES*

H. L. Berk, L. D. Pearlstein and G. M. Walters

Lawrence Radiation Laboratory, University of California

Livermore, California

The negative mass instability has generally been studied in the absence of or in the zero temperature limit of a plasma background. We have extended the negative mass formalism to treat beams immersed in plasmas that have Larmor radii comparable to the beam Larmor radius. A new dissipation or inverse dissipation then arises since the beam excites the natural modes of oscillation of the background plasma whose group velocity is outward. This new effect has been applied to mirror machine experiments such as Alice I and Phoenix I where the magnetic field decreases in the radial direction. In this case, due to the dielectric properties of the plasma, the standard negative mass mode arising from particles concentric with the axis of the machine is a stable wave. However, instability results from the new mechanism when the coupling of the concentric negative mass particles to the stable loss-cone modes is considered.

Further, it is postulated that the saturated non-linear modes can continually extract energy from the negative energy plasma background and expand radially. This would explain the continual decrease in frequency of the instability that is observed in experiments.

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*Work done under the auspices of the U.S. Atomic Energy Commission.

To be presented at the Annual CTR Theory Meeting, Gatlinburg, Tennessee, April 24 - 25, 1969.

STABILIZATION OF NEGATIVE ENERGY EIGENMODES IN MIRROR GEOMETRY*

L. D. Pearlstein and H. L. Berk

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Livermore, California

In a previous talk it was pointed out[1] that the stable (in infinite homogeneous geometry) negative energy waves[2] inherent to loss-cone distributions become "absolutely" unstable when the effects of inhomogenities along the magnetic field are included. At that time only conditions for instability were discussed.

Here via a WKB technique, wherein both turning and singular points occur, scale length criteria for stabilization are presented. Roughly speaking, the origin of stabilization is due to the negative dissipation, arising from the singularities due to the cyclotron resonances, which competes with the positive dissipation arising from the outward flow of energy characteristic of these eigenmodes. Results are compared with an exact numerical solution of the governing differential equation. It is seen that except for the lowest eigenmode where WKB is least accurate agreement is excellent.

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*Work done under the auspices of the U. S. Atomic Energy Commission.

[1] H. L. Berk, L. D. Pearlstein, J. Corday, J. D. Callen, W. C. Horton and M. N. Rosenbluth, Bull. Am. Phys. Soc. 13 1492 (1968).

[2] It should be pointed out that at non-zero parallel wavelengths these are the unstable convective loss-cone modes.

To be presented at the Annual CTR Theory Meeting, Gatlinburg, Tenn., April 24-25, 1969.

PULSE PROPAGATION IN AN INHOMOGENEOUS CONVECTIVELY UNSTABLE PLASMA*

Laurence S. Hall

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Livermore, California

New techniques for the determination of the response to a pulse disturbance of an inhomogeneous or time-varying unstable medium[1] are applied to the study of an inhomogeneous convectively unstable plasma. Two simple limiting cases are studied, (a) the Alfven wave firehose instability and (b) the high-density cyclotron-resonant (electron-ion) instability. In both instances, the inhomogeneity is chosen to be appropriated to mirror geometry. The conditions under which reflections confine the instability, causing unbounded growth at finite spatial positions (essential instability) are found and discussed.

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*Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] L. S. Hall, Pulse Propagation in Inhomogeneous Time-Varying Unstable Media, Bull. of the APS 13, 1492 (1968).

To be presented at the Annual CTR Theory Meeting, Gatlinburg, Tennessee, April 24 - 25, 1969.

PARTICLE MOTION IN THE PRESENCE OF MULTIPLE FINITE AMPLITUDE HARMONIC CYCLOTRON MODES AND THE NON-LINEAR DYNAMICS OF RESONANT LOSS-CONE INSTABILITIES*

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ABSTRACT

The dynamics of particles moving in the presence of multiple, finite amplitude, longitudinal flute modes with frequencies near a harmonic of the cyclotron frequency is discussed. It is shown that two finite amplitude waves with different wavelengths can cause groups of hot particles to move coherently for long periods of time. This coherent motion then breaks the hot particle constraints found in the single wave picture(1) and can induce nonlinear instabilities. The effect of this mechanism is evaluated for hot-cold resonant loss cone instabilities and saturation effects and amplitudes are estimated.

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* Work supported in part by the U.S. Atomic Energy Commission.

(1) R.E. Aamodt and S.E. Bodner, Bull.Am.Phys.Soc., 13, 1528 (1968).

WAVE-WAVE INTERACTIONS, WAVE PARTICLE SCATTERING, AND EXPLOSIVE INSTABILITIES*

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Knoxville, Tennessee

and

E. G. Harris
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and

Oak Ridge National Laboratory
Oak Ridge, Tennessee

We have used quantum mechanical techniques to calculate the matrix elements for some non-linear processes in a plasma in a magnetic field. The waves considered are transverse waves propagating parallel to the field (Alfven waves, whistlers, etc.), longitudinal waves propagating parallel to the field (plasma oscillations, ion sound waves), and longitudinal waves propagating perpendicular to the field. Matrix elements have been calculated for three-wave interactions and wave-particle scattering. These matrix elements may be used to write kinetic equations for waves and for particles.

If the plasma particles have a loss cone distribution then the three-wave interaction can give rise to an explosive instability in which the amplitude of a wave becomes infinite in a finite time.[1] The three waves considered here are longitudinal waves propagating perpendicular to the field. For a special case of two positive energy and one negative energy waves, we found that the time required for the waves to grow to infinite amplitude was given by

$$t_{\infty} \simeq \frac{5.5}{\omega_p}\left(\frac{\eta}{\delta\eta}\right)$$

where ω_p is the plasma frequency, η is the unperturbed density, and $\delta\eta$ is the initial perturbation in density due to one of the waves.

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* Research at the University of Tennessee was supported by the U.S. Atomic Energy Commission under contract AT-(40-1)-2598. Research at ORNL was sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

1 M. N. Rosenbluth, B. Coppi, and R. N. Sudan, Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, CN-24/E-13, 1968 (to be published).

UNSTABLE PLASMA WAVES PROPAGATING PERPENDICULAR TO A MAGNETIC FIELD: SMALL AMPLITUDE GROWTH AND NONLINEAR SATURATION FROM COMPUTER EXPERIMENTS*

C. K. Birdsall,† J. A. Byers, D. Fuss and M. Grewal

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Livermore, California

Computer experiments have followed the growth of the instability for a uniform plasma with $f(v_\perp) = \delta(v_\perp - v_o)$ in a uniform magnetic field. The growth rates and frequencies from linear analysis[1,2] have been checked to within a few percent over as much as 25 orders of growth, for a variety of modes and parameters (such as ka_i and $(\omega_{pi}/\omega_{ci})^2$, in both one and two-dimensional models. This wide range is due to using noise-free starting conditions. The electric field energy growth stops (saturates) at about one percent of the total energy, with recurring peaks (bursts) dependent on the density of modes involved. The major spreading of $f(v_\perp)$ occurs in the last cyclotron period before saturation, as will be shown in a movie of v_x-v_y space. Saturation appears to be roughly coincident with $f(v_\perp)$ spreading sufficiently for no linear growth at the $(\omega_{pi}/\omega_{ci})^2$ used. Using broader distributions, we have found that the marginal densities and velocity spreads required for stability to be in agreement with linear theory. Addition of a small amount of cold plasma to an otherwise stable hot distribution causes instability with similar growth rates. A nonlinear instability has also been observed.

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*Work done under the auspices of the U. S. Atomic Energy Commission.

†Permanent Address: Elec. Engr. and Computational Sciences Dept., University of California, Berkeley, Calif.

[1] R. A. Dory, G. E. Guest and E. G. Harris, Phys. Rev. Ltrs. 14, No. 5, 131 (1965).

[2] F. W. Crawford, J. A. Tataronis, J. of Appl. Phys. 36, 2930 (1965)

To be presented at the Annual CTR Theory Meeting, Gatlinburg, Tenn., April 24-25, 1969.

ON THE REVERSIBILITY OF THE GYRORESONANCE IN MIRROR MACHINES

E. Canobbio[+] - G. Ichtchenko

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Département de Physique du Plasma et de la Fusion Contrôlée
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It has been proposed that high frequency fields can confine thermonuclear plasmas in magnetic mirrors /1/. Recently the "exact resonant" approach to confinement has been criticized /2/ and replaced by a "nearly resonant" approach in which no H.F. power is dissipated in the plasma.

We show in this paper that there are particles which are reflected after crossing the exact resonance surface without a net change in magnetic moment , and that the mean energy of most of the trapped particles is bounded . The mean energy is of the order of magnitude of the energy gained in a single crossing of the resonance surface by a particle initially at rest. The analytical conditions for a reversible single crossing and for the quasi-periodicity of the trapped particle motion are established . Extensive numerical calculations support the analytical results. We suggest that the H.F. power dissipated in the exact resonant approach to confinement can be reduced when such effects are taken into account.

[+] EURATOM-CEA Association.

/1/ T. Consoli, Paper CN-24/J 1, Novosibirsk Conference (1968)

/2/ C. J. H. Watson, 2ème Colloque Interactions Champs Oscillants et Plasmas, Saclay, (1968), Vol. II.

SESSION II

April 24, 2:00 p.m. — Chairman, R. L. Morse

Toroidal Equilibria Including $\underset{\sim}{E} \times \underset{\sim}{B}$ and Parallel Plasma Flow,*

S. Yoshikawa, Princeton University. -- Toroidal devices of closed magnetic field lines have equilibria other than governed by the condition $p = p(\oint d\ell / B)$ if the kinetic energy of the plasma perpendicular to the magnetic field becomes comparable with the potential energy associated with the magnetic well.[1] A system with shear has only equilibria with $p = p(\psi)$ where ψ is the magnetic surface. However, the time required to reach this equilibrium from the time of the initial plasma injection is finite. Calculations show that for a particular model, the equilibrium times are in the vicinity of several times the Bohm time. Implication of $\underset{\sim}{E} \times \underset{\sim}{B}$ motion to stability will also be considered (i.e., Kelvin-Helmholz instability).

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* This work was performed under the auspices of the U.S. Atomic Energy Commission.

[1] S. Yoshikawa and M.R. Barrault, Princeton Plasma Physics Laboratory MATT-626 (1968).

Diffusion in Toroidal Plasmas with Radial Electric Field.

T.E. Stringer

(U.K.A.E.A., Culham Laboratory, Berkshire, England).

The equilibrium of an axi-symmetric toroidal plasma is analysed by expanding in the powers of the inverse aspect ratio. In an earlier treatment of the equilibrium of a resistive plasma by Pfirsch and Schlüter the density variation over magnetic surfaces was neglected, while in a recent analysis of a weakly collisional plasma by Galeev and Sagdeev the potential variation over surfaces was neglected. In this more general treatment the variation in all parameters is included.

In the resistive case the density variation becomes important in the presence of a zero order radial potential distribution $\Phi_o(r)$. It leads to a large enhancement in the equilibrium diffusion when the doppler shifted frequency seen by the rotating plasma, $-(1/rB)\,\partial\Phi_o/\partial r$, approaches the frequency of a natural mode in the plasma (drift or ion-acoustic mode).

In the weakly collisional case the neglect of $\Phi_1(r,\theta)$ is inconsistent with quasi-neutrality. It couples the ions and electrons, and for general $\Phi_o(r)$ it leads to a diffusion rate for both species comparable to the faster of the two rates obtained by Galeev and Sagdeev. In practice $\Phi_o(r)$ will build up to a distribution such that the diffusion rate of the two species are everywhere equal. In typical experimental conditions the subsequent ambipolar rate is less than the general level, and may be comparable to the slower of the rates derived by Galeev and Sagdeev.

PRL 22, 770 (1969)

Effect of Trapped Particles on Stellarator Equilibria,* D. R. Dobrott, J. M. Greene, and J. L. Johnson,† Princeton University. -- The stellarator expansion provided a formalism for determining toroidal equilibrium configurations on a guiding-center model in which there is no electrostatic potential along field lines.[1] For more general distributions, particles trapped in the mirrors associated with the small-amplitude helical field interact with the electrostatic field in a manner analogous to the nonlinear electrostatic oscillations of Bernstein et al.[2] The equilibrium is specified by the distribution functions $f^{\pm}(\Psi, \mu, \mathcal{E})$ for the untrapped particles, some general conditions on the trapped particles, the net parallel current on each magnetic surface, and boundary conditions on the magnetic fields. The problem is reduced to a set of partial differential equations.

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* Work performed under the auspices of the U.S. Atomic Energy Commission.

† On loan from Westinghouse Research Laboratories.

[1] D. R. Dobrott and J. L. Johnson, Plasma Physics (in press); MATT-633.

[2] I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).

Numerical Simulation of Toroidal Low-β Confinement with a Fluid Model,* N. K. Winsor, J. L. Johnson,† and J. M. Dawson, Princeton University. -- A numerical program for studying toroidal confinement[1] has provided some understanding of quasi-static plasma diffusion when the potential due to electron pressure gradients along the lines and Hall currents are ignored. Inclusion of these terms introduces significant deviations from the standard theory. The results of calculations made with zero resistivity look qualitatively like those of Smith and Bishop, but differ quantitatively. In typical calculations, with parameters corresponding roughly to the model C stellarator with a constant rotational transform, the plasma density shows quite significant variations on a magnetic surface (about ten per cent). Loss times scale with magnetic field, temperature, and rotational transform classically (using the Pfirsch-Schlüter factor), but the absolute value is about five times faster.

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* Work performed under the auspices of the U.S. Atomic Energy Commission. Use was made of Computer Facilities supported by the National Science Foundation Grant NSF-GP 579.

† On loan from Westinghouse Research Laboratories.

[1] J. L. Johnson, N. K. Winsor, and J. M. Dawson, Bull. Am. Phys. Soc. II, 13, 1539 (1968).

Equilibrium Electric Fields in Axisymmetric Toroids,* C. G. Smith, Princeton University. --An equilibrium characterized by potential surfaces which do not coincide with the magnetic flux surfaces has been previously observed in the numerical simulation of axisymmetric toroidal confinement.[1] Specifically, the potential well was shifted toward the inside (i. e., toward the major axis). As the model did not include collisions, the parallel electric field could not have been the result of resistive effects. The observation may be explained, however, by the following two-step theory. First, the electron motion along the field lines, on a fast time-scale, is such as to try to maintain charge neutrality at all times. Since the potential is (algebraically) greatest where the electrons are concentrated, this then will also be where the ions are concentrated. Second, the ions, on a long time scale, will eventually shift to the outside to be in toroidal equilibrium. Therefore, the potential should be greatest to the outside or, conversely, the well should be to the inside. Although Poisson's equation was used in the numerical work to determine the potential, the first step was seen to be instantaneously satisfied during the approach to equilibrium. The stationary potential variation along field lines is such that, for the electrons, the potential reinforces and is of the same magnitude as the mirroring force due to the toroidal structure of the magnetic field. Hence the effective mirroring boundary in phase space is considerably altered from the picture previously held, and the expectation is that Galeev-Sagdeev banana diffusion is enhanced.

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] C. G. Smith and A. S. Bishop, IAEA Conf. on Plasma Physics, Paper NC-24/D-9, Novosibirsk.

Critical β from Stellarator and Scyllac Expansions

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New York University

Abstract

The Scyllac expansion[1] differs from the Stellarator expansion[2] in two major respects, the scaling of β and the pressure profile. A flat pressure profile minimizes the magnetic field pathology near the axis and allows explicit calculations. Used in the low β Stellarator expansion, the flat profile gives improved stability, allowing $\beta_c \sim 1/2$ or larger in some parameter ranges.

Using the finite β Scyllac expansion, a stable range $\beta_1 < \beta < \beta_2$ near $\beta \sim 3/4$ is found for $\ell = 1$ and small kr_0 because of unusual wall effects.[3] The plasma can be centered with respect to the wall without affecting stability; there is no relation between m = 1 stability and any equilibrium plasma shift toward the wall. These helical m = 1 modes can easily be stabilized with a small z-current, but unstable kink modes then arise.

Because of nonuniform limits, the lower (Stellarator) critical β is invisible using the Scyllac expansion, and the entire wall stabilization effect is missing in the Stellarator expansion.

[1] A.A. Blank, H. Grad, H. Weitzner, 'Toroidal High-β Equilibria,' To appear in Proceedings of IAEA Conference at Novosibirsk, August 1968.

[2] J. Greene, J. Johnson, K. Weimer, Plasma Physics 8, 145 (1966).

[3] H. Weitzner, invited address at Miami, November 1968.

S. Fisher, B. Marder and H. Weitzner
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New York University

Abstract

In order to obtain the equilibrium of an E layer in an Astron device easily by numerical computation, we have chosen the simple model used by many authors.[1] We assume axial symmetry, a non-relativistic rigid rotor distribution function for electrons, and stationary ions that exactly balance the electron space charge. The stream function for the magnetic field satisfies the differential equation in dimensionless variables

$$\frac{1}{r}\frac{\partial^2\psi}{\partial z^2} + \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r} = re^{-\psi+cr^2} ,$$

where c measures the ratio of the rotational energy of the electrons to their thermal energy. We solve for the combination $\chi = \psi - cr^2$ and give plots of level lines of $\chi(r,z)$.

The obvious iteration scheme, which puts ψ_n into the right hand side and then solves the differential equation for ψ_{n+1}, converges only up to the first point of bifurcation. The results compare well with the z independent solution, and various plots are given. In order to obtain solutions past the point of bifurcation and with much deeper wells we go over to an entirely different mode of computation and obtain solutions as convergent power series. We find that it is easy to obtain equilibria with deep wells. We finally examine other possible computational schemes to go past the points of bifurcation.

[1] G. Benford, D.L. Block, N.C. Christofilos, T.K. Fowler, V.K. Neil, L.D. Pearlstein, 'Stability of E-Layer and Plasma in Astron Configuration'. Paper CN24-F10, Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, USSR, August 1-7, 1968.

Hydromagnetic Plasma Confinement In a Spherical Geometry

G.K. Morikawa and E. Rebhan
Courant Institute of Mathematical Sciences
New York University

Abstract

Toroidal configurations for plasma confinement in a spherical geometry are constructed. A one-parameter family of exact hydromagnetic equilibrium solutions over the range of the pressure parameter, $0 \leq \beta \leq 1$, are found yielding a continuous family of plasma equilibria from the force-free limit ($\beta=0$) up to maximum pressure, $\beta=1$. Some stability considerations are discussed.

SESSION III

April 25, 9:00 a.m. – Chairman, E. G. Harris

NON-LINEAR EVOLUTION OF THE TWO STREAM INSTABILITY*

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ABSTRACT

We consider the streaming instability of a cold electron beam travelling with velocity u through a cold electron plasma, where the ratio of beam density to plasma density is much smaller than one. Within the framework of a single wave theory[1], for which beam particle trapping provides the mechanism of nonlinear growth stabilization, the existence of a unique nonlinear BGK stationary state to which the system can evolve is established. The electrostatic field energy of this stationary state is also evaluated.

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(1) W.E. Drummond and John Malmberg, Bull.Am.Phys.Soc. 13, 1542 (1968)

* Work supported in part by the U.S. Atomic Energy Commission

ABSTRACT

Two-Dimensional Numerical Simulation of Warm Two-Beam Plasma.* R. L. Morse and C. W. Nielson. Los Alamos Scientific Laboratory. Two-dimensional simulations of longitudinal electrostatic instability development in warm two-beam plasma have been run and compared with the same cases in one dimension. In the bump-on-tail case, the overshoot of total electrostatic field energy develops in 2d as in 1d, but then the energy falls abruptly to the thermal level instead of showing the slow decrease seen in 1d. The same is true in the beam and cold background case where it is also observed that the beam is more effectively slowed by the background in 2d, than in 1d. In all cases the vortex structure which is so prominent in phase space in 1d starts to form but quickly dissipates. This latter result is best seen in the two-dimensional case of two equal warm beams.

*Work performed under the auspices of the U. S. Atomic Energy Commission

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NON-LINEAR DAMPING OF A FINITE AMPLITUDE MONOCHROMATIC WAVE

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In this work we study the time evolution of the amplitude of a large amplitude monochromatic Langmuir wave. We take into account non-linear effects due to particle trapping. The average distribution function is computed neglecting the time variation of the electric field amplitude and the harmonics of the wave as in [1]. The conservation of total energy yields the equation of evolution for the wave amplitude. We determine the mean variation of this amplitude and we show that it is not directly related to the ratio of linear daming rate to trapping frequency. This effect is a consequence of the large variation of the average distribution function for particles which are rather far from resonance. The importance of initial conditions for the perturbed distribution function is stressed.

[1] L.M. AL'TSHUL, V.I. KARPMAN, Soviet Physics JETP 22 (1966), p. 361

On-Line Plasma Simulation

Burton D. Fried, University of California, Los Angeles and TRW Systems,
Chuan Sheng Liu, University of California, Los Angeles
and R. Z. Sagdeev, Institute of Nuclear Physics, Novosibirsk

We report here some initial experiments on the use of the UC Santa Barbara mathematical on-line system for plasma simulation. Since such problems typically involve very lengthy computer runs, batch computing methods have always been used heretofore and the value of on-line access to the computer is not clear a priori. However, in many problems, a large fraction of the computing is devoted to following the trajectories of particles which do not play an essential role in the development of the system and which could be treated in a less rigorous fashion without greatly affecting the character of the solution. Our strategy is therefore to follow, on-line, only those particles which, due to resonances or the like, do have an essential effect upon the behavior of the system, treating the remainder in some appropriate, approximate way (e.g., using a fluid model or the solution of the linearized kinetic equation). Once the physics of the situation is clear, a batch calculation in which all particles are followed numerically can be undertaken. As a first experiment along this line, we have studied the interaction of resonant particles with a single electrostatic wave, an old problem which in the small amplitude limit simply gives Landau damping. A linearized treatment is used for the non-resonant particles; energy conservation is used to determine the change in the wave-amplitude; and the motion of the resonant particles is followed exactly. Thus, larger amplitude waves than that required for validity of Landau's theory can be studied; although the restriction $e\Phi << kT$ must still be imposed, we can allow the bounce frequency, $\omega_b = (eEk/m)^{1/2}$, to be comparable to the Landau damping rate, γ_L, and thus obtain results outside the region of validity of either Landau's theory ($\gamma << \omega_b$) or O'Neil's analysis (which assumes $\gamma << \omega_b$). The effects of collisions among the resonant particles can also be observed by including, in the equation of motion, sheet crossing terms of the Dawson type.

ABSTRACT

Numerical Simulation of a Fire Hose Type Instability.* D. O. Dickman, R. L. Morse, and C. W. Nielson. Los Alamos Scientific Laboratory. A cylindrical, axisymmetric sheet pinch with reversed B_z (the Astron geometry) and $T_{\parallel} >> T_{\perp}$ has been simulated numerically. The simulations show unstable growth of a hose-like mode in the r, z cross section, in general agreement with linear theory. Under some conditions, the large amplitude growth of this mode is arrested by compression of the magnetic flux between the plasma and an outer conducting wall or the center axis, after which the plasma bounces toward the initial equilibrium position. Changes in the dominant z wavelength have been observed at this point. This process is seen to be accompanied by some transfer of particle energy from $T_{\parallel}$ to $T_{\perp}$.

*Work performed under the auspices of the U. S. Atomic Energy Commission

Numerical Study of Finite β Drift Waves

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Abstract

Drift waves in a plane with uniform magnetic field $B_z(x,y,t)$ are calculated numerically. We consider only finite β waves, since electric field effects and finite β effects are decoupled in this geometry. The wave behavior is studied in equilibria which are stable, and in equilibria which are unstable (on the basis of linear theory). A comparison will be made of the particle method and of the difference method of numerical solution of the equations.

A Trapped Particle Instability,* W. L. Kruer and J. M. Dawson, Princeton University. -- A simple model for a new instability resulting from particles trapped in a large amplitude wave has been invoked to explain the generation of satellite frequencies in a recent experiment by Wharton, Malmberg, and O'Neil (the WhaMO experiment). This model predicts the growth of satellites on the large amplitude wave at a frequency separation proportional to the $\sqrt{E}$, where E is the large wave amplitude. We have investigated this instability on a 1-D sheet model. A large amplitude electrostatic wave is produced by driving an initially Maxwellian plasma, and the evolution of the wave spectrum is followed. The energy in modes with wave numbers less than that for the large amplitude mode exponentiates over a factor of roughly 100. The modes with higher wave numbers grow less, but these modes would appear to be damped. The major features observed in this calculation are in semi-quantative agreement with those of the WhaMO experiment and with the predictions of the theory.

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* This work was performed under the auspices of the U. S. Atomic Energy Commission. Use was made of Computer Facilities supported by the National Science Foundation Grant NSF-GP 579.

Compression of an Axially Symmetric Plasma with Non-Isotropic Pressure

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Abstract

A two-fluid collisionless model of a plasma is used to describe the behavior of the pinch, that is, a fully ionized gas under axially symmetric radial compression. Two pressure tensors are derived and used in the equations, one for the ions and one for the electrons. The ion pressure is determined by taking higher moments of the ion Vlasov equation while the electron terms are found from an asymptotic expansion of the electron distribution function.

The equations of motion are put into Lagrangian coordinates and solved numerically on the CDC 6600 digital computer.

In all pinches we considered , we observe the familiar magnetic compression waves associated with the two-fluid model. A train of waves is generated at the boundary and propagates inward as the radius of the plasma column decreases. The pulses peak strongly upon reaching the center and are then reflected outward where they interfere with and pass through the remaining incoming tail of the wave train.

One pinch considered is a hybrid z-theta pinch in which a plasma which contains a constant axial magnetic field is subjected to a rising circumferential field on the boundary. As the plasma column contracts an increasing axial field is applied at the boundary in such a manner as to preserve the initial axial magnetic flux in the gas. It is found that in this way strong compressions in which the average density increases to greater than 25 times its initial value can be obtained without the solutions becoming discontinuous.

In all cases the compressions are sufficiently weak so that the solutions are well behaved, that is no shocks develop.

SESSION IV

April 25, 2:00 p.m. — Chairman, T. K. Fowler

NON-LINEAR EVOLUTION OF A SINGLE ELECTROSTATIC ELECTRON-ION STREAMING UNSTABLE MODE

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ABSTRACT

The non-linear evolution of a single mode[1] of an electrostatic electron-ion instability[2] due to relative streaming across the magnetic field is determined. The relevance of this instability to the generation of turbulence in the normal hydromagnetic shock is discussed.

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1) R.E. Aamodt and S.E. Bodner, Bulletin of the American Physical Society, Vol. 13, 1528 (1968).

2) H. Vernon Wong, Bulletin of the American Physical Society, Vol. 13, 1563 (1968).

Amplitude Limitation and Transport for Collisional Drift Waves*

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and

F. L. Hinton
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Collisional drift waves in the regime of the Hendel, Chu and Politzer[1] experiment are investigated using the finite Larmor radius fluid equations and neglecting heat transport across the magnetic field. The equations are solved for the amplitude versus B dependence of the single mode solutions using the perturbation theory of Simon[2] for nonlinear transport. The amplitude of the three dimensional mode is limited principally by the strong collisional damping at the first harmonic component. The anomalous transport is calculated and is compared with the experimental observations of transport and wave amplitude in the Princeton Q-1 device.

1. H. W. Hendel, T. K. Chu, and P. A. Politzer, Phys. Fluids 11, 2426(1968).

2. A. Simon, Phys. Fluids 11, 1181(1968).

*Work performed under the auspices of the U. S. Atomic Energy Commission.

Time and Space Correlation Due to Wave-Particle Scattering

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Department of Physics, and
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Turbulent diffusion is usually described in terms of its effect on the average density $\langle n \rangle$. However by studying the same process in terms of $\langle nn \rangle$, one obtains a picture of blobs or eddies which on the average diffuse down the density gradient. The blobs, whose sizes are initially of the order of a wavelength, are constantly sheared apart by the waves, producing progressively smaller blobs in a fashion reminiscent of hydrodynamic turbulence. Motion along the field lines tends to mix the eddies and smooth them out.

The harmonic spectrum produced is considerably different than the usual three wave resonant coupling between propagating modes. Instead, the coupling is via wave-particle scattering to ballistic or equilibrium modes. The resulting spectrum has a power law dependence of both ω and k and might explain the occurrence on such spectra in a variety of experiments.

This work was supported principally by the National Science Foundation Grant GK 2581

Electrical Conductivity of Plasma in a Spatially Inhomogeneous Magnetic Field,* F. L. Hinton and C. Oberman, Princeton University.--The electrical conductivity of plasma in a strong spatially inhomogeneous magnetic field has been calculated, by using the small gyroradius approximation to the kinetic equation. The current which flows along the magnetic lines is the sum of two parts, one due to the time-independent applied electric field, and the other (secondary currents) associated with gradient-B drifts across the lines. The first part gives the electrical conductivity. It has a nonlocal dependence on the electric field, when the mean free path is not short compared with the scale length for magnetic field variations along the field lines. The kinetic equation is solved by a perturbation technique in the long-mean-free-path limit, and an explicit expression for the conductivity is obtained. When the magnetic field variation is small the conductivity is reduced from the constant field case approximately by the fraction of magnetically trapped particles. The resistance of a stellarator plasma column is found to be roughly 30 per cent greater than the classical value, when the mean free path is not short. Although the calculation was carried out for a Lorentz plasma, we expect the result to be approximately correct for a real plasma.

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* This work was performed under the auspices of the U.S. Atomic Energy Commission, and was partially funded by the Air Force Office of Scientific Research.

(Abstract of a ten-minute paper submitted for presentation at the Sherwood Theory meeting, to be held in Gatlinburg, Tennessee, April 24 and 25, 1969. Paper to be Presented by J. A. Wesson).

Ion Drift Wave Instability.* J. P. Freidberg and J. A. Wesson, Los Alamos Scientific Laboratory. In mhd unstable shearless plasmas the flute mode, $k_{\parallel} = 0$, is stabilized for sufficiently large $k_{\perp}$ by finite Larmor radius effects. It has been generally assumed that the case $k_{\parallel} = 0$ is the worst mode and therefore the case $k_{\parallel} \neq 0$ has received little consideration. A preliminary investigation of the case $k_{\parallel} \neq 0$ using the Vlasov equation indicates that there is a solution $\omega_i = 0$ (time dependence $e^{(i\omega_r + \omega_i)t}$) for a certain $k_{\parallel_c} > 0$. The question arises as to whether this is a marginally stable situation. If so then modes having any value of $k_{\perp}$ would be unstable for $k_{\parallel}^2 < k_{\parallel_c}^2$. This would appear to contradict FLR theory for $k_{\parallel} = 0$. The apparent contradiction has been investigated and the outcome is as follows. Instability occurs for all $0 < k_{\parallel} < k_{\parallel_c}$. For those $k_{\perp}$ which are unstable for $k_{\parallel} = 0$ the instability for $k_{\parallel} \neq 0$ is essentially the mhd mode reduced in growth rate by finite Larmor radius effects and line bending. For those $k_{\perp}$ which are stable for $k_{\parallel} = 0$ instability at finite $k_{\parallel}$ is due to ion resonance with the ion drift wave. The growth rate of this instability increases with β. For sufficiently low β it is stabilized by electron Landau damping.

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

VLASOV DESCRIPTION OF A PURE ELECTRON PLASMA IN A MAGNETIC FIELD

By

Nicholas A. Krall and Ronald C. Davidson

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A variety of fusion schemes (e.g. Astron, Electrostatic confinement) may involve a knowledge of the stability behavior of non-neutral plasmas. In view of this and of other recent experimental studies[1] we discuss the ultimate un-neutralized plasma, the collisionless electron gas in a magnetic field. Basic properties of this system are derived from the Vlasov equations, including (a) self-consistent equilibria including space charge, and (b) associated dispersive properties and stability behavior. Vlasov equilibria which are rigid rotar (in the mean) about the confining field, and which correspond to electron density profiles which are essentially uniform within the interior of the electron gas column are constructed. The associated electrostatic dispersion relation is derived (including space charge effects) for "body" waves interior to the column. The resulting dielectric function is similar in structure to the corresponding results for a neutral plasma, and an elemental algorithm is given for obtaining electron gas stability information from neutral plasma literature.

[1] A. W. Trivelpiece, R. E. Pechacek, and C. A. Kapentanakos, Phys. Rev. Letters 21, 1436 (1968).

Dispersion Relation for Electron Oscillations,* R. G. Bateman and M. D. Kruskal, Princeton University. -- In previous plasma kinetic theories, the effects of Landau damping, collisional damping, damping due to time variation of the distribution function, etc., have been calculated separately as independent phenomena. Here, we derive a unified dispersion relation (for uniform plasma with no magnetic field) to describe the properties of collective oscillations. It is found that a uniform electric field further alters the form of the dispersion relation. The altered imaginary part of the dispersion relation is particularly important in calculations of the resonant contributions to the Balescue-Lenard equation as well as in the calculation of wave damping. For electron oscillations with large phase velocities, the complete damping predicted by the unified dispersion relation is not uniformly approximated by the (Landau) damping predicted by the ordinary dispersion relation. The derivation may be illustrated with the nonlinear Vlasov equation. After multiple-time-scaling and iteration, all terms proportional to the potential ϕ are collected before the equation is solved for f and ϕ. The coefficient of ϕ goes into the diepersion relation. The Klimontovich formalism is used for more complete analysis.

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* This work was performed under the auspices of the U.S. Atomic Energy Commission, and was partially funded by the Air Force Office of Scientific Research.

Laser Induced Anomalous Heating of a Plasma,* P. Kaw and J. M. Dawson, Princeton University. -- When a thin metal film or a solid pellet is irradiated with an intense laser pulse, it is converted into a blob of dense plasma. Because of the violent manner in which this plasma is produced, large nonthermal density fluctuations may be excited in it. Further, if the laser pulse is sufficiently intense, it drives one or both of the following two low frequency instabilities in the plasma: (a) Thermal Instability: this arises if energy is fed into a certain region of the plasma at a rate higher than that at which it can be diffused away by thermal conduction and other dissipative processes. (b) Parametric Instability: this arises if energy is fed parameterically into plasma waves and ion acoustic waves at a rate higher than that at which they damp away. These low frequency instabilities can amplify the initial fluctuations in the plasma to produce large amplitude ion waves. Such ion waves in turn can lead to considerable enhancement of the high frequency resistivity of the plasma, especially around the plasma frequency. The laser beam can thus cause an anomalous heating of the plasma. Typical estimates show that laser powers currently being used are sufficiently high to cause this effect to be important.

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* This work was performed under the auspices of the U. S. Atomic Energy Commission.

Papers Not Scheduled for Oral Presentation

Dilip K. Bhadra, "Trapped Particle Effect on the Drift-Cyclotron Instability in a Multipole Machine".

S. Fisher and H. Grad, "Finite Beta Equilibria".

J. M. Greene, J. L. Johnson, and K. E. Weimer, "Effect of Longitudinal Currents on a Toroidal Theta Pinch".

W. L. Kruer and J. M. Dawson, "Small Amplitude Failure of Quasi-Linear Theory".

J. Marsh and P. H. Rutherford, "Anomalous Reflection and Transmission of Cyclotron Waves".

F. Winterberg, "Ignition of Thermonuclear Micro-Explosions by Intense Relativistic Electron Beams".

Trapped Particle Effect on the Drift-Cyclotron Instability in a Multipole Machine[†]

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The effect of trapped particles on the drift-cyclotron instability in a multipole machine has been considered. Dispersion equations have been obtained for the propagation of this instability in a nonuniform collisionless plasma using a simplified approach which takes into consideration the magnetic field strength and curvature variation along field lines. The results seem to show the existence of a branch of oscillations with frequency $\omega \sim \bar{\Omega}_i - k_\perp \bar{V}_g - \bar{\omega}_b$, where $\bar{\Omega}_i$, $\bar{V}_g$ and $\bar{\omega}_b$ are the average gyrofrequency, drift velocity due to magnetic field gradient, and ion 'bounce' frequency, respectively, $k_\perp$ being the wave-vector transverse to the magnetic field. This may explain the discrepancy between the experimentally observed frequency and the theoretical value obtained previously. This branch of the oscillation also shows a higher growth rate.

[†]The work reported herein was supported by the U.S. Atomic Energy Commission under Contract No. AT(04-3)-167, Project Agreement 38.

Finite Beta Equilibria

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Abstract

Numerical methods are utilized to calculate finite beta equilibria in cusped and mirror geometry. The flux surfaces and constant magnetic field surfaces are obtained by solving the guiding-center fluid equation

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} + \frac{1}{\sigma}\left(\frac{\partial\psi}{\partial r}\frac{\partial\sigma}{\partial r} + \frac{\partial\psi}{\partial z}\frac{\partial\sigma}{\partial z}\right) = -\frac{r^2}{\sigma}\frac{\partial P_{\parallel}}{\partial\psi} \tag{1}$$

where $\sigma = 1 + \frac{P_{\perp} - P_{\parallel}}{B^2}$. The properties of the solution to equation (1) are exhibited in terms of the distribution function, vacuum magnetic field configurations and the boundary conditions. The maximum allowable beta in a long thin mirror machine is determined as a function of the mirror ratio to simulate a theta-pinch experiment. Analytic calculations for low beta equilibria show that plasmas in cusps and mirrors can be globally para-magnetic. Finite beta equilibria which are globally paramagnetic are being looked for numerically.

Effect of Longitudinal Currents on a Toroidal Theta Pinch,* J.M. Greene, J.L. Johnson,† and K.E. Weimer, Princeton University. -- Recently much effort has been devoted to the theoretical problems of producing a toroidal theta pinch equilibrium. Stellarator theory indicates that the rotational transform provided by a helical field could produce such an equilibrium,[1] but the unfavorable line curvature of such a system limits the contained pressure to a very low value.[2] Here we discuss the effect of adding a longitudinal current in the plasma surface.[3] This greatly increases the rotational transform, and thus the stabilizing effect of conducting walls. Bobeldijk[4] has shown that the associated kink instabilities can be suppressed by introducing a force free field with constant rotational transform into the region between the plasma column and the walls.

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* This work was performed under the auspices of the U.S. Atomic Energy Commission.

† On loan from Westinghouse Research Laboratories.

[1] A.A. Blank, M. Grad, and H. Weitzner, Third Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, USSR, (1968) paper CN-24/K6.

[2] M.N. Rosenbluth, J.L. Johnson, J.M. Greene, and K.E. Weimer, Phys. Fluids (in press); MATT-665.

[3] F. Ribe and W.B. Riesenfeld, Plasma Physics 11, 2035 (1968).

[4] C. Bobeldijk, Rijnhuizen Report 68-45 (1968).

Small Amplitude Failure of Quasi-Linear Theory,* W. L. Kruer and J. M. Dawson, Princeton University. -- The breakdown of quasi-linear theory for large wave amplitudes is well-known. However, there is also a small amplitude failure of the theory for finite size systems. This is due to the breakdown of the continuous wave number approximation implicit in the theory. In more physical terms, particles must be able to come into resonance with various modes of the wave spectrum; i.e., to 'hop' from one mode to another. The condition that they be able to do this is roughly that $2v_{TR} > \Delta(\omega/k)$, where v_{TR} is the trapping velocity in a typical wave trough $(= 2\sqrt{eE/mk})$ and $\Delta(\omega/k)$ is the gap in phase velocity between adjacent modes due to the finite size of the system. We have numerically investigated the motion of resonant particles in a noisy electric field. The computer results confirm quasi-linear theory for amplitudes where it is expected to apply and confirm its breakdown at the expected small and large amplitude limits. The low amplitude limit can be significant when the system contains only a few large amplitude waves (wavelength roughly equal to the size of the system).

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* This work was performed under the auspices of the U. S. Atomic Energy Committion. Use was made of the Computer Facilities supported by the National Science Foundation Grant NSF-GP 579.

Anomalous Reflection and Transmission of Cyclotron Waves,* J. Marsh and P. H. Rutherford, Princeton University. -- A transverse cyclotron wave, frequency ω, propagating along a decreasing magnetic field is classically totally absorbed where $\omega = \Omega$, where Ω is the gyro frequency. An investigation has been made into anomalous reflection and transmission processes, involving trapped particle effects, for such a wave propagating into a magnetic mirror. Using an approach similar to that of Berk et al[1] for the case of electrostatic waves, expressions have been found for the anomalous reflection and transmission coefficients. The condition $\oint (\Omega - \omega) d\ell / v_{\parallel} = n\pi$ selects those particles which contribute to the effect, and phase integral methods enable the coefficients to be evaluated. In typical cases the effect is small.

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* This work was performed under the auspices of the U.S. Atomic Energy Commission.

[1] H. L. Berk, C. W. Horton, M. N. Rosenbluth, D. G. Baldwin, and R. N. Sudan, Phys. Fouids 11, 365 (1968).

Ignition of Thermonuclear Micro-Explosions by Intense Relativistic Electron Beams

F. Winterberg

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The different methods for igniting a small thermonuclear explosion by irradiating a target with a beam of energetic particles are discussed with regard to their relative merits and promise of success. On the basis of this analysis, the scheme in which the target heating is effected by an intense relativistic electron beam seems particularly promising as a method for achieving the desired goal. Beams of the required intensity can be produced with high efficiency by the high voltage Marx-circuit Blumleinline technique. A second method using a charged, levitated, highly magnetized superconducting ring may produce electron beams of substantially higher voltage and total energy output than in the Marx-circuit technique. The target heating is brought about by collective plasma instabilities. In case the collision-free beam dissipation should pose unforeseen difficulties, it is shown that one may alternatively irradiate the thermonuclear target by an intense beam of ions. In this case the stopping power range ensures complete collisional beam energy dissipation. A further distinct advantage of the methods described is the reduction of the range of the fusion products by the strong self-magnetic field of the intense electric current, accompanied by a reduction of the critical ignition size, by which it may even be possible to extract energy from a D-D thermonuclear reaction. The energy produced by a chain of thermonuclear micro-explosions occurring inside a spherical container can be converted into useful electrical energy.

A modification of this scheme can be used for rocket propulsion by having the explosions take place at the center of a spherical reflector open on one side.